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On the value and stability of some exact steady state solution of some problems in the transient micro-wave heating of ionized media

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Abstract : The value and stability of the exact steady state solution of the average electron energy balance equation are investigated for three different kinds of homogenuous plasmas heated by a microwave field. These are, a weakly ionized plasma, a strongly ionized plasma and a hydrogen plasma in which the collision cross section is approximated by an analytical expression which fits experimental data. The effect of the field frequency and field amplitude on the value and stability of the steady state average electron energy as well as its limiting values are investigated for the three cases considered.

Keywords : Microwave heating, ionized media, exact steady state, stability, electron energy.

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1. Introduction

A plasma is a medium subject to different kinds of instabilities. Terminology and nomenclature of plasma instabilities are numerous and confusing. In this paper, we investigate the stability and the value of the steady state average electron energy of 3 different kinds of homogenuous plasmas heated by a microwave field. These are, a weakly ionized plasma, a strongly ionized plasma and an atomic hydrogen plasma in which the collision cross section of electrons is approximated by an analytical expression which fits experimental data. We assumed that all electron generation and loss processes are neglected and that electron collisions are elastic. The fraction of electron energy lost per collision is considered to be constant. For the power absorbed by an average electron from the field, we used an average value. These assumptions were considered by several investigators to study different phenomena either exactly or numerically. Ginzburg (1964) studied the steady-state characteristics of the interaction of a homogenuous plasma and a strong uniform electric field. Numerical solutions of the non linear model of this interaction for the build-up of the electron energy and electron density when a step sinusoidal high power RF field is applied to a weakly ionized medium was investigated (Gould and Roberts 1956, El-Khamy *et al* 1970 and El-Khamy 1974). The same assumptions were used in the study of the problem of third harmonic generation of electromagnetic waves in weakly and strongly ionized homogenuous collisional plasmas (Sodha *et al* 1975) and also, in the study of the number of the steady state value of the average electron energy of a homogenuous strongly ionized plasma, heated by a microwave field (Negm and El-Khoga 1988). In this paper we have investigated the effect of the field frequency and field amplitude on the value and stability of the steady state average electron energy of the 3 different kinds of homogenuous plasmas already mentioned. The effect of the field amplitude is found to be the same on the 3 kinds of plasma studied. However, the field frequency has proved to have a different effect on the different kinds of plasma studied. Meanwhile, we have found that the 3 kinds of plasma behaved in the same way when the field frequency is increased without limit.

In Section 2, we have presented the general basic equations. In Sections 3, 4 and 5 we have presented the equations as well as the results of the investigation of the first, second and third kind of plasma respectively. Discussion and conclusion of the results of Sections 3, 4 and 5 are given in Section 6.

2. Basic equations

We assumed that a step sinusoidal microwave field $E(t)=E_0 \sin wt$ has been applied to an ionized medium at the instant $t=0$. The magnitude of the amplitude of the field E_0 is large enough to cause appreciable heating of the electrons but is not so large to make changes in the electron density of the medium. All processes of electron loss are also neglected. Under these simple assumptions, the response of the electrons to this field is governed by the equations of motion and temperature (Ginzburg 1964, Sodha *et al* 1975 and Gurevich 1978).

$$\frac{dv}{dt}=\frac{-eE}{m}-\nu_0 v \tag{1}$$

$$\frac{3}{2}\frac{dT_e}{dt}=-eEv-\delta\nu_0\frac{3}{2}(T_e-T_0) \tag{2}$$

where T_0 is the temperature of the heavy particle and assumed constant ; T_e is the average temperature of the electrons at time t ; at $t=0$, T_e is assumed to be equal to T_0 ; e, m, v and ν_0 are the electron charge, mass, velocity and collision frequency respectively, δ is the fraction of electron energy lost in a collision with the heavy particle. For an elastic collision, δ is constant and is equal to $\frac{2m}{M}$ where

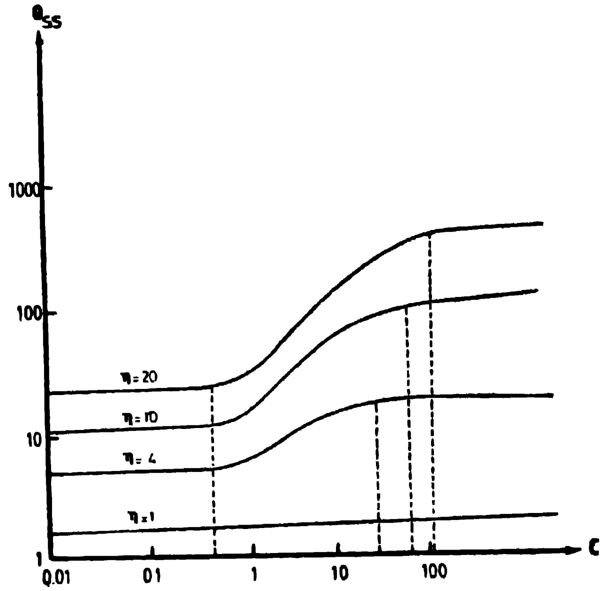

Figure 2. Variation of the steady state average nomalized electron temperature θ_{ss} of a weakly ionized plasma with the nomalized field frequency c at different values of normalized field amplitude η.

$$\nu_c = \nu_{co}\left(\frac{T_e}{T_0}\right)^{-} \tag{10}$$

Substituting from eqs. (3) and (10) in eq. (2) one obtains

$$\frac{d\theta}{d\tau} = \frac{[c^2\theta^4 - (\Gamma^2 + c^2)\theta^3 + \theta - 1]}{3/2(1 + c^2\theta^3)} \tag{11}$$

where

$$\Gamma = \eta(1 + c^2)^{\frac{1}{2}}$$

The exact solution of eq. (11) has been found by El-Khoga and Negm (1988) and the variation of θ with τ has been calculated for different values of c and η. Also, the number N of the steady state values of θ as well as the corresponding steady state values of θ have been investigated by (Negm and El-Khoga 1988) and (Negm and El-Khoga 1989) respectively.

As in the case of a weakly ionized plasma, the zeros of the characteristic equation that determine the stability criterion are obtained from eq. (8) but here $f(\theta)$ is the right hand side of eq. (11). λ is found to have the value

$$\lambda = \frac{[3(\Gamma^2 + c^2)\theta^2_{ss} - 4c^2\theta^3_{ss} - 1]}{\theta^{3/2}_{ss}(1 + c^2\theta^3_{ss})} \tag{12}$$

It is easily seen that the sign of λ is the same as that of the numerator of this equation, When $N=1$ (Negm and El-Khoga 1988) we found that λ is $-$ve and θ_{ss} is stable. Figure 3 shows the variation of θ_{ss} with η at different values of c. In

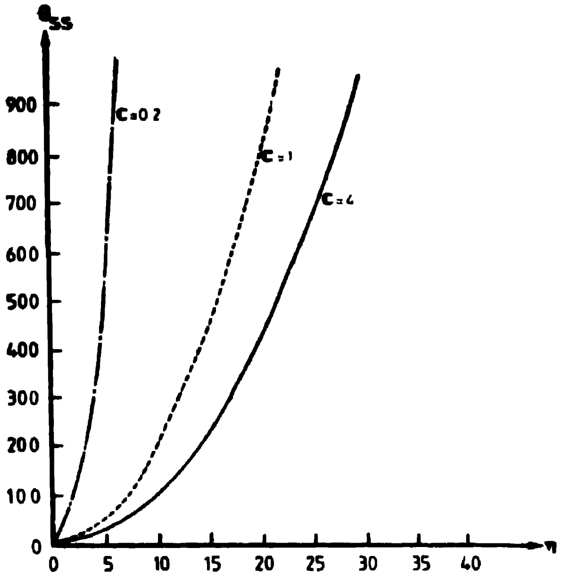

Figure 3. Same as Figure 1 but for a strongly ionized plasma.

contrary with the behaviour of a weakly ionized plasma, θ_{ss} decreases with increase of c. From Figure 4 we see that this decrease occurs for low values of c.

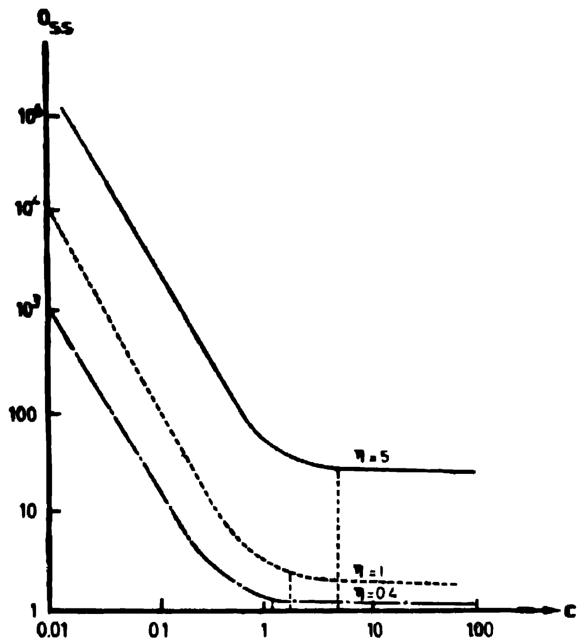

Figure 4. Same as Figure 2 but for a strongly ionized plasma.

From Figure 4, we can notice also that there is a value c_{cri} of c, above which the increase of c has no effect on θ_{ss}, we can also notice that the value of c_{cri} increases with the increase of η. Negm and El-Khoga (1988) found that when c and η are both $<$ 1, N may be equal to 3 and we found that not all the three values of θ_{ss} are stable. We found that if θ_1, θ_2 and θ_3 are three steady state values of θ such that $\theta_1 < \theta_2 < \theta_3$ then both θ_1 and θ_3 are stable but θ_2 is unstable. Figure 5 shows

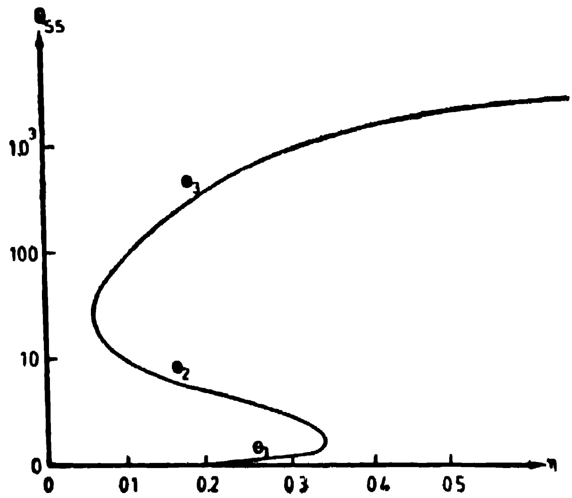

Figure 5. Variation of the two stable steady state average normalized electron temperature θ_1 and θ_2 and the unstable state average normalized electron temperature θ_3 with η when c=0.01.

the variation of the two stable steady state average normalized electron temperature θ_1 and θ_3 and the unsteable one θ_2 with η when $c=0.01$. The results presented in Figures 6 and 7 are the same as those of Figure 5 but for $c=0.1$ and 0.2

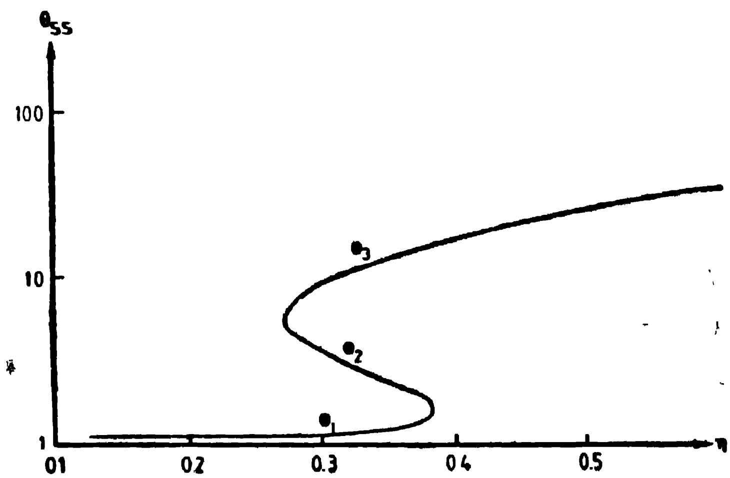

Figure 6. Same as Figure 5, but for c=0.1.

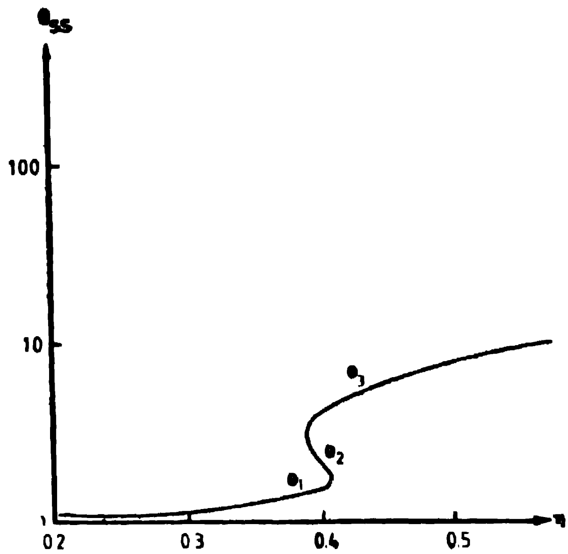

Figure 7. Same as Figure 5, but for $c = 0.2$.

respectively. From these Figures, both θ_1 and θ_3 increase with the increase of η and with the decrease of c, while θ_2 has a different behaviour, it decreases with the increase of η and the decrease of c.

5. Values and stability of the steady state average electron temperature of a hydrogen plasma-collision cross section is approximated from experimental results

However, when the plasma is neither weakly nor strongly ionized, the theoretical calculation of ν_0 is very complicated.

Usually, for this case ν_0 is estimated from experimental measurements of the value of the collision cross section σ_0. The experimental results of Brackman and Fite (1958) for the dependence of σ_0 on the electron energy in an atomic hydrogen plasma can be approximated by a simple relation in which σ is inversely proportional to electron energy (El-Khoga 1975). The corresponding dependence of ν_0 on T_e was found (El-Khoga 1975) to be

$$\nu_e = \nu_{c0} \left(\frac{T_e}{T_0}\right)^{-\frac{1}{2}} \tag{14}$$

For this case, we have to use the eqs. (14), (3) and (2) to get the energy balance equation. This has the form

$$\frac{d\theta}{d\tau} = -\frac{[c^2\theta^{3/2} + (1 - \Gamma^2 - c^2)\theta^{\frac{1}{2}} - \theta^{-\frac{1}{2}}]}{(1 + c_\theta^2)} \tag{15}$$

For this case N=1 and θ_{ss} has the value

$$\theta_{ss}=\frac{1}{2c^2}\{(\Gamma^2+c^2-1)+[(\Gamma^2+c^2-1)^2+4c^2]^{\frac{1}{2}}\} \tag{16}$$

Figure 8 shows the variation of θ_{ss} with η at different values of c. We have found that the differences between the two curves of c=1 and c=4 are too small. Thus

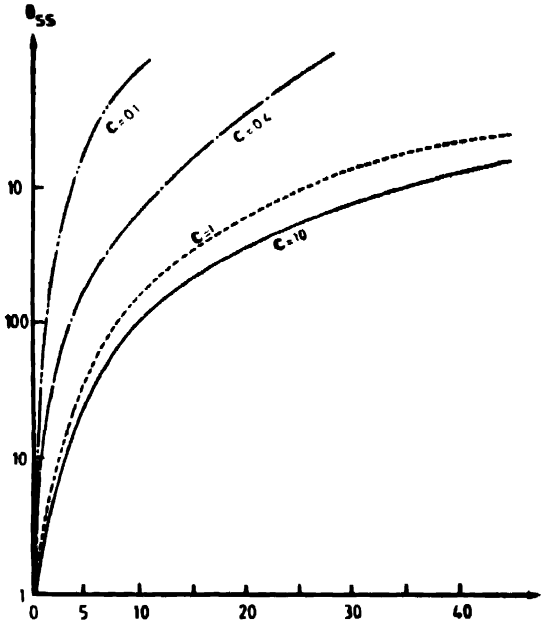

Figure 8. Same as Figure 1 but for a plasma in which the dependence of the collision cross section on electron energy fits experimental data.

the variation of the frequency has a smaller effect on the heating process of this plasma so long as c is $>$ 1. When c is $<$ 1, θ_{ss} decreases with the increase of c. This result is presented in Figure 9.

6. Discussion and conclusion

Comparing Figures 2, 4 and 9 we may conclude that the increase of c above a critical value has no effect on the value of θ_{ss} for the three kinds of plasma studied here. This critical value of c for any of the three kinds of plasma depends on the value of η. Also, it is simply found from eqs. (6), (13) and (16) that for the three kinds of plasmas studied

$$\lim_{c\to\infty}\theta_{ss}=1+\eta^2 \tag{17}$$

Eq. (17) agrees with Ginzburg's equation for high frequency heating of a fully ionized medium. Thus, we prove that this equation is to be satisfied also for high

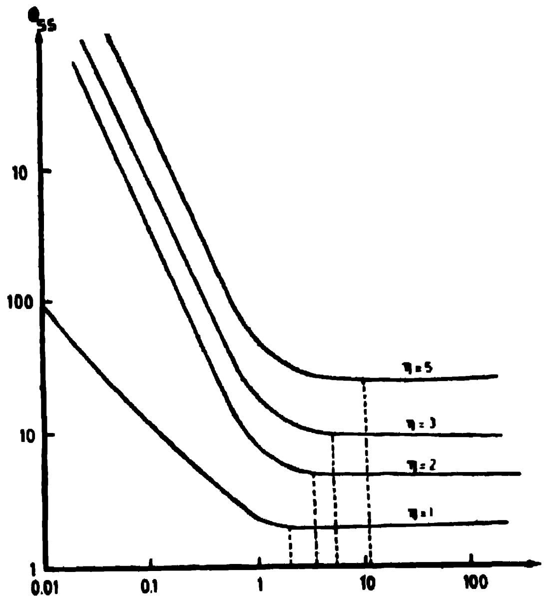

Figure 9. Same as Figure 2 but for a plasma in which the dependence of the collision cross section on electron energy fits experimental data.

frequency heating of a weakly ionized medium and a medium in which the dependence of σ_0 on the electron energy fits experimental data. Also we can conclude that the stability of the heating process by a microwave field is ensured for the first and third kind of plasma studied here. The stability of the heating process of the strongly ionized plasma is satisfied when $N=1$. The case in which $N \neq 1$ may occur only when $c < 1$, thus the necessary and sufficient condition for N to be equal to 1 when $c < 1$ found by Negm and El-Khoga (1988) can also be considered as a necessary and sufficient condition for the stability of θ_{ss} when $c < 1$. This condition has the value

$$\eta < \eta_{cri} = \frac{c^{2/3}\,(1-c^{2/3})^{\frac{1}{2}}}{(1+c^2)^{\frac{1}{2}}} \tag{18}$$

Moreover when $c < 1$, the necessary but not sufficient condition for N to be equal to 3 found by the same authors and given by

$$\eta > \eta_{cri} \tag{19}$$

can be considered as a necessary but not sufficient condition for the instability of the steady state electron temperature of a strongly ionized homogenuous plasma heated by a microwave field.

Finally, it is quite important to mention that the above results are obtained with the assumptions mentioned in Section 2. The assumption of constant electron density is justified in a strongly ionized plasma, in the weakly and third kind of plasma studied, the variation of the electron density can be neglected so long as the electron temperature or energy is below the threshold energy for ionization I (El-Khoga and Negm 1987).

In the cases in which the variation of the electron density must be considered, the response of the electron to the microwave field will be governed by three equations ; namely, equation of conservation of mass in addition to equation of motion (1) and equation of energy or temperature (3). This system of 3 equations can not be reduced to a single non-linear autonomous differential equation like the three ones studied here in eqs. (5), (11) and (15).

Also, the assumption that δ the fraction of electron energy lost in a collision with the heavy particle is constant, is justified so long as the collision is elastic (Ginzburg 1964). Fortunately, the inelastic collisions are expected to occur when the electron energy exceeds a threshold value (Mott and Massey 1965) when this assumption is not justified and the dependence of δ on T_e must be considered. In this case, the system of equations can be reduced to a single non-linear autonomous differential equation like the one shown in eq. (5), (11) and (15), but may be of more complicated form. Hence, we may conclude that the results obtained are valid, since the assumptions considered are justified.

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